Assignment #1

Date Due: October 14, 2024

Total: 100 marks

We have the following languages:
$L_1 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that begin with 1010$\}$,
$L_2 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that end with 01011$\}$,
$L_3 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with 010 being a subword$\}$,
$L_4 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with an odd number of 0's$\}$,
$L_5 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with an even number of 2's$\}$,
$L_6 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having the forth symbol from the right end a 0$\}$,
$L_7 = \{$the set of all strings over the alphabet $\{0,1,2\}$ beginning with 12022$\}$,
$L_8 = \{$the set of all strings over the alphabet $\{0,1,2\}$ ending in 12022$\}$,
$L_9 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with the number of 1's multiple of 6$\}$,
$L_{10} = \{$the set of all strings over the alphabet $\{0,1,2\}$ with the number of 1's multiple of 5$\}$,
$L_{11} = \{$the set of all strings over the alphabet $\{a,b\}$ with the number of a's multiple of 7$\}$,
$L_{12} = \{$the set of all strings over the alphabet $\{a,b\}$ with the number of b's multiple of 6$\}$,
$L_{13} = \{$the set of all strings over the alphabet$\{0,1,2\}$ consisting only of alternating groups of 10 and 01 (10 and 01 *alternates* at least once)$\}$,
and the following homomorphisms
$h : \{a,b\} \longrightarrow \{0,1,2\}^*$, $h(a) = 01$, $h(b) = 21$; and $g : \{0,1,2\} \longrightarrow \{a,b\}^*$, $g(0) = a$, $g(1) = ba$, $g(2) = \varepsilon$.

1. (maximum 25 marks) Compute the languages (5 marks each)
   (a) $L_{20} = L_1 \cap L_2$.
   (b) $L_{21} = 01011\Sigma^* \cap \Sigma^*1010$
   (c) $L_{22} = L_{13}$
   (d) $L_{23} = L_6$
   (e) $L_{24} = L_7 \cap L_8$
   (f) $L_{25} = L_{11} \setminus L_{12}$
   (g) $L_{26} = h^{-1}(L_4)$
   (h) $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$
   (i) $L_{28} = g(L_1^R)$
2. (maximum 55 marks, 10 marks each) For each of the following languages give a DFA accepting it over the alphabet $\{0,1,2\}$ or $\{a,b,c\}$, depending on the alphabet of the language.

(a) L_{20}
(b) L_{21}
(c) L_{22}
(d) L_{23}
(e) L_{24}
(f) L_{25}
(g) L_{26}
(h) L_{28}

3. (20 marks) Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 1, 2, 4, 6\}$:

   (a) the set of all strings beginning with a 1, 2 or 4, that, when the string is interpreted as an integer in base 7, is a multiple of 4 plus 1. For example:
   - strings 1,41,210,221,2061,2010, 2612, 202012,102642, and 440614 are in the language;
   - the strings 2, 4, 01, 21, 212, 610, 0221, 4062,4021,6014, and 035 are not.

   (b) The set of all strings that ends with an **1, 2, or 4** and when the string is interpreted ***in reverse*** as an integer **in base 7, is a multiple of 4 plus 1**.
   - Examples of strings in the language are 1,14,012,122,1602,0102,2162, 210202, 246201, and 416044.
   - Examples of strings that are not in the language are: 2, 4, 10, 12, 212, 016, 1220, 2604 , 1204, 4106 , and 530.

4. (10 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow 0$	1	0
1	2	1
$* 2$	3	2
3	1	3

   Informally describe, as simple as possible, the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

The maximum is bounded to 115 marks.

Very Important: Verify your solutions using Grail (5 marks for each of exercises 2,3, and 4); describe ***how do you think*** for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to ***all*** exercises in class.